

Practice Final Exam

You do not need to show your work when you row reduce a matrix. You may use the `rref()` command in your calculator.

1. a) Solve the system of equations. Write your answer in vector notation.

$$3x_2 - 6x_3 + 6x_4 + 4x_5 = -5$$

$$3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 = 9$$

$$3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 = 15$$

- b) Determine whether the solution set is a subspace of \mathbb{R}^5 .

2. Let $A = \begin{bmatrix} 3 & -2 \\ 3 & -4 \end{bmatrix}$.

a) Find the characteristic polynomial of A.

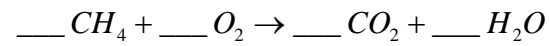
b) Find the eigenvalues of A.

c) Find the eigenvector for each eigenvalue of A.

d) Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

e) Use your answer from part d), find the general form for A^k .

3. Methane (CH_4) burns in oxygen (O_2) to produce carbon dioxide(CO_2) and water (H_2O). Balance the chemical equation, using the smallest whole number solution:



4. The matrix below is an augmented matrix from a system of equations with 3 variables and 4 equations. Find the values of h for which the system is consistent.

$$A = \begin{bmatrix} 2 & 1 & 1 & -2 \\ -4 & -4 & 1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 2 & -3 & h \end{bmatrix}$$

5. Let $\mathbf{u} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$.

a) Compute $(\mathbf{u} \cdot \mathbf{v}) \mathbf{v}$

b) Find $\|\mathbf{u}\|$

c) Find a unit vector in the direction of \mathbf{u} .

d) Find the angle between \mathbf{u} and \mathbf{v} .

e) Find the orthogonal projection of \mathbf{v} onto \mathbf{u} .

6. Let $A = \begin{bmatrix} 3 & 1 & 1 \\ -2 & 0 & 0 \\ -3 & 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 0 & 1 \\ 2 & -2 & 1 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$.

a) Find $A\mathbf{x}$.

b) Find AB .

c) Find $(A + B)\mathbf{x}$.

7. Determine whether each of the following is a subspace of \mathbb{R}^3 . If it is, find a basis.

$$\text{a) } W = \begin{bmatrix} 2a - b \\ -6a + 3b \\ 4b - 2a \end{bmatrix}$$

$$\text{b) } H = \begin{bmatrix} a - b \\ a + b \\ 2a + 2 \end{bmatrix}.$$

8. Let $\mathbf{b}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

a) Find the coordinates of the vector $\mathbf{x} = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$.

b) Find the vector \mathbf{x} with coordinates $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$.

9. a) Find a basis for the span of the polynomials.

$$\{1 - 4x^2, 3x + 4x^2, 2x, 3 - 4x + 2x^2\}$$

b) Find the coordinates of the polynomial in the basis found in part a)

$$p(x) = 5x^2 + 2x - 3$$

10. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be the linear transformation defined by $T \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a+b \\ c \end{bmatrix}$.

a) Find the matrix A that corresponds to the transformation.

b) Find a basis for $\text{Nul}(A)$.

c) Find a basis for $\text{Col}(A)$.