Math 284
Cuyamaca College

Name:___________________
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Practice Final Exam

You do not need to show your work when you row reduce a matrix. You may use the rref() command in your calculator.

1. a) Solve the system of equations. Write your answer in vector notation.

$$\begin{aligned} 3x_2 - 6x_3 + 6x_4 + 4x_5 &= -5 \\ 3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 &= 9 \\ 3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 &= 15 \end{aligned}$$

b) Determine whether the solution set is a subspace of $\mathbb{R}^5$.

2. Let $A=\begin{bmatrix} 3 & -2 \\ 3 & -4 \end{bmatrix}$.

a) Find the characteristic polynomial of A.

b) Find the eigenvalues of A.

c) Find the eigenvector for each eigenvalue of A.

d) Find an invertible matrix P and a diagonal matrix D such that $A=PDP^{-1}$.

e) Use your answer from part d), find the general form for A^k.

3. Methane (CH_4) burns in oxygen (O_2) to produce carbon dioxide(CO_2) and water (H_2O). Balance the chemical equation, using the smallest whole number solution:

$$\_\_\_ CH_4 + \_\_\_ O_2 \rightarrow \_\_\_ CO_2 + \_\_\_ H_2O$$

4. The matrix below is an augmented matrix from a system of equations with 3 variables and 4 equations. Find the values of h for which the system is consistent.

$$A = \begin{bmatrix} 2 & 1 & 1 & -2 \\ -4 & -4 & 1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 2 & -3 & h \end{bmatrix}$$

5. Let $\mathbf{u} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$.

a) Compute $(\mathbf{u} \bullet \mathbf{v})\,\mathbf{v}$

b) Find $\|\mathbf{u}\|$

c) Find a unit vector in the direction of $\mathbf{u}$.

d) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

e) Find the orthogonal projection of $\mathbf{v}$ onto $\mathbf{u}$.

6. Let $A = \begin{bmatrix} 3 & 1 & 1 \\ -2 & 0 & 0 \\ -3 & 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 0 & 1 \\ 2 & -2 & 1 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$.

a) Find $A\mathbf{x}$.

b) Find AB.

c) Find $(A + B)\mathbf{x}$.

7. Determine whether each of the following is a subspace of $\mathbb{R}^3$. If it is, find a basis.

a) $W = \begin{bmatrix} 2a-b \\ -6a+3b \\ 4b-2a \end{bmatrix}$

b) $H = \begin{bmatrix} a-b \\ a+b \\ 2a+2 \end{bmatrix}$.

8. Let $\mathbf{b}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

a) Find the coordinates of the vector $\mathbf{x} = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$.

b) Find the vector $\mathbf{x}$ with coordinates $[x]_{\mathcal{B}} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$.

9. a) Find a basis for the span of the polynomials.

$$\left\{1-4x^2,\ 3x+4x^2,\ 2x,\ 3-4x+2x^2\right\}$$

b) Find the coordinates of the polynomial in the basis found in part a)

$$p(x)=5x^2+2x-3$$

10. Let $T:\mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by $T\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a+b \\ c \end{bmatrix}$.

a) Find the matrix *A* that corresponds to the transformation.

b) Find a basis for Nul(A).

c) Find a basis for Col(A).